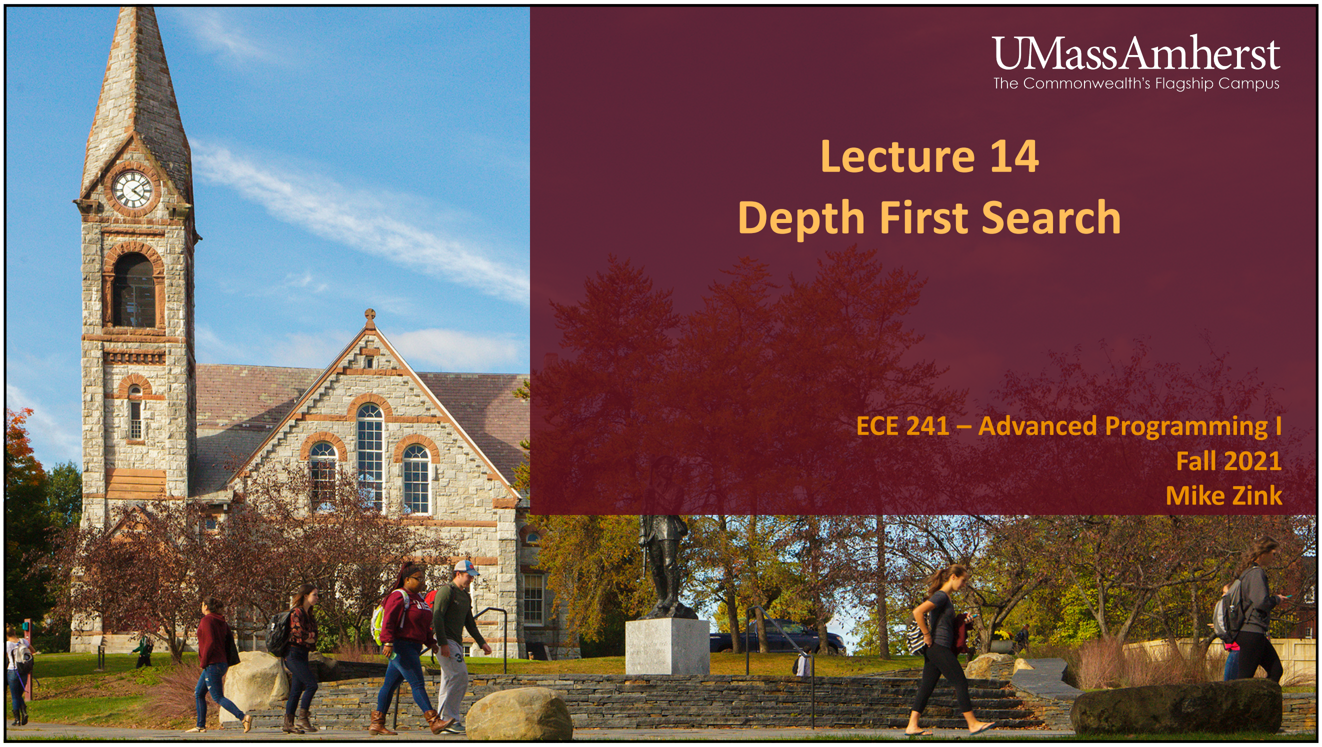

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The Commonwealth's Flagship Campus

Lecture 14
Depth First Search

ECE 241 – Advanced Programming I
Fall 2021
Mike Zink

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Overview

- Depth First Search
- Topical sorting

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Objective

- Understand and be able to apply the depth first search (DFS) algorithm
- Apply Topological Sorting as graph algorithm

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Knights Tour Problem

- Puzzle played on chess board with single figure, the knight
- Objective: find sequence of moves that allow knight to visit every square on board "exactly" once
- Such sequence is called "tour"
- Upper bound on possible tours is $1.35 * 10^{35}$
- Use graph search to solve problem

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Knights Tour Problem

Solve problem by using two main steps:

- Represent legal moves of knight on chessboard as graph
- Use a graph algorithm to find path of length *rows*×*columns*−1 where every vertex on graph is visited exactly once

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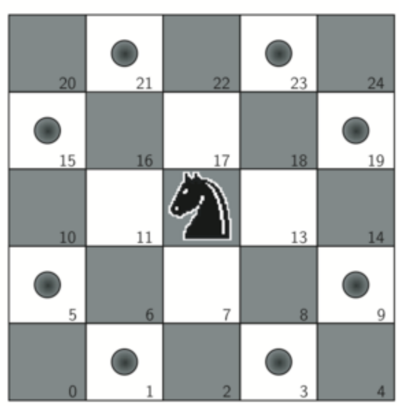
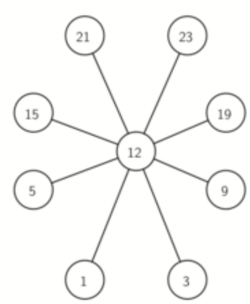
Knights Tour Problem

- Each square represented as node in graph
- Each legal move represented by edge





Building the Graph

```
from Graph import Graph

def knightGraph(bdSize):
    ktGraph = Graph()
    for row in range(bdSize):
       for col in range(bdSize):
           nodeId = posToNodeId(row,col,bdSize)
           newPositions = genLegalMoves(row,col,bdSize)
           for e in newPositions:
             nid = posToNodeId(e[0],e[1],bdSize)
                ktGraph.addEdge(nodeId,nid)
    return ktGraph

def posToNodeId(row, column, board_size):
    return (row * board_size) + column
```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Building the Graph

```
def genLegalMoves(x,y,bdSize):
    newMoves = []
    moveOffsets = [(-1,-2),(-1,2),(-2,-1),(-2,1),
                   ( 1,-2),( 1,2),( 2,-1),( 2,1)]
    for i in moveOffsets:
       newX = x + i[0]
       newY = y + i[1]
         if legalCoord(newX,bdSize) and \
                     legalCoord(newY,bdSize):
             newMoves.append((newX,newY))
    return newMoves

def legalCoord(x,bdSize):
    if x >= 0 and x < bdSize:
       return True
    else:
         return False
```

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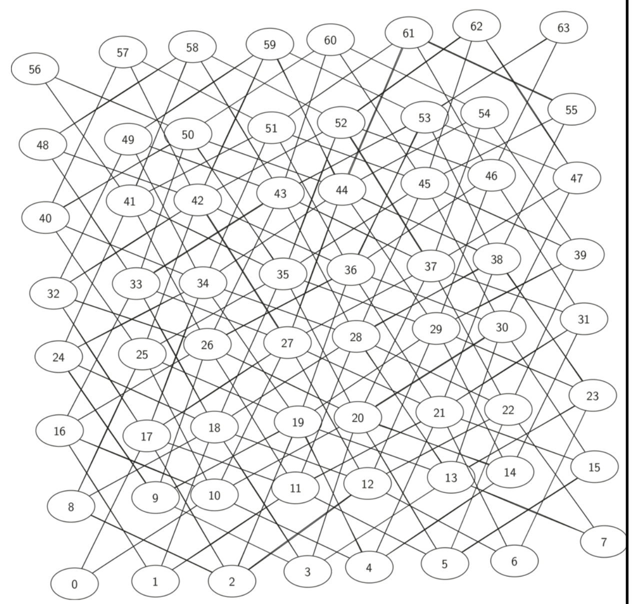
Complete Graph

- 336 edges
- Less connections for vertices on edges of board
- Sparsity:
  - Fully connected graph: 4096 egdes
  - Matrix only 8.2% filled



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Depth First Search (DFS)

- Solve problem width depth first search (DFS) algorithm
- Creates search tree by exploring one branch of the tree as deeply as possible
- We will look at two algorithms:
  1. Directly solves problem by explicitly forbidding a node to be visited more than once
  2. More general, but allows nodes to be visited more than once as the tree is constructed

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Implementing Knight's Tour

- DFS exploration of graph finds path with exactly 63 edges
- When dead end is found (more moves possible)
  - Algorithm backs up tree to next deepest vertex allowing a legal move

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knighTour - Function

```
from Graph import Graph, Vertex

def knightTour(n,path,u,limit):
        u.setColor('gray')
        path.append(u)
        if n < limit:
            nbrList = list(u.getConnections())
            i = 0
            done = False
              while i < len(nbrList) and not done:
                if nbrList[i].getColor() == 'white':
                    done = knightTour(n+1, path, nbrList[i], limit)
                i = i + 1
              if not done:  # prepare to backtrack
                path.pop()
                u.setColor('white')
        else:
            done = True
        return done
```

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DFS – Coloring

- DFS uses colors to keep track which vertices have been visited
  - White: unvisited
  - Gray: visited
- If neighbors of particular vertex have been explored && length of vertices < 64 => dead end reached
- If dead end reached => backtrack (Return from `knightTour` wirh `false`)

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DFS – Coloring

- Since DFS is recursive, use stack to help with backtracking
- After return from `knightTour` with status `False`:
  - Remain inside while loop
  - Look at nextvertex in `nbrlist`

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Simple Example

- Following figures show steps of search
- Assume `getConnections` orders nodes in alphabetical order
- Start with calling `knightTour(0,path,A,6)`

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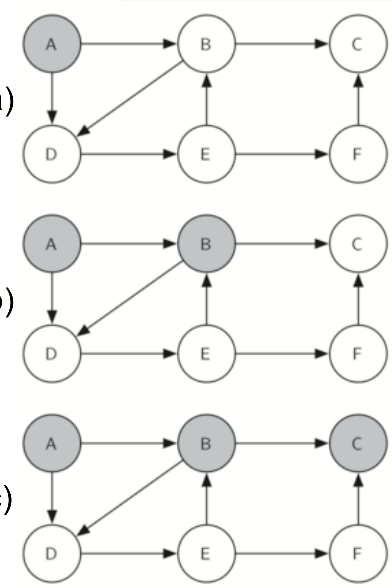
Simple Example

- `knightTour` starts with node A (a))
- B and D are adjacent to A
- Since B comes next in alphabet, it is chosen next (b))
- Recursively calling `knightTour` explores B

a)

b)

c)



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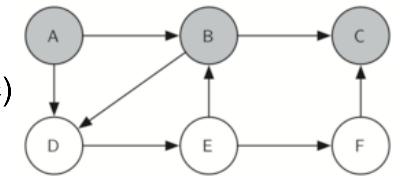
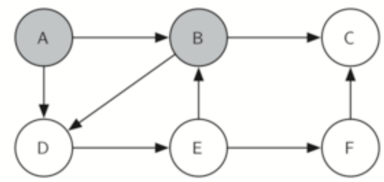
Simple Example

- B is adjacent to C and D
- `knightTour` elects to explore C
- C is dead end with no adjacent white notes (c))
- Change color of C back to white (d))
- Backtracks search to vertex B

c)



d)



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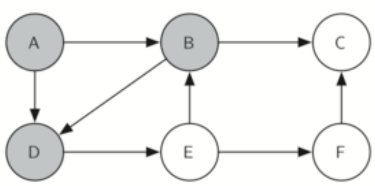
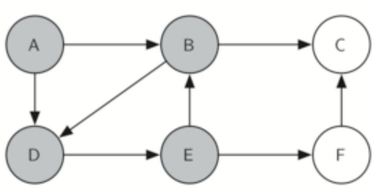
Simple Example

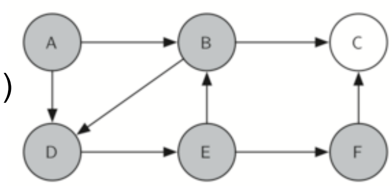
e)



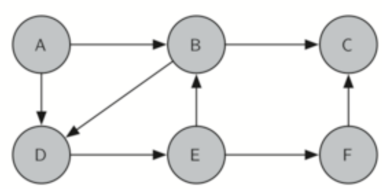
f)



g)



h)



- Next vertex to explore is D (e))
- `knightTour` makes recursive calls until we get to node C again (f), g), h))

Simple Example

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- when we get to node C the test `n < limit` fails
- => all nodes in graph exhausted
- return `True` to indicate that we have made a successful tour of the graph
- return the list, `path` has the values `[A,B,D,E,F,C]`, which is the the order we need to traverse the graph to visit each node exactly once

Simple Example

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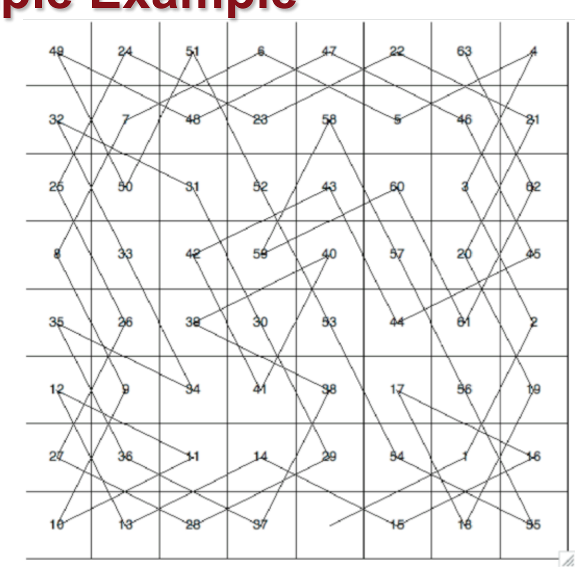

- Complete tour around 8 x 8 board

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Knight's Tour - Analysis

- Very sensitive to method used to select next vertex
- Example
  - 5 x 5 board, calculate path in 1.5 second
  - 8 x 8 board, up to ½ hour
- Reason: $O(k^N)$, N is number of squares, k is small constant

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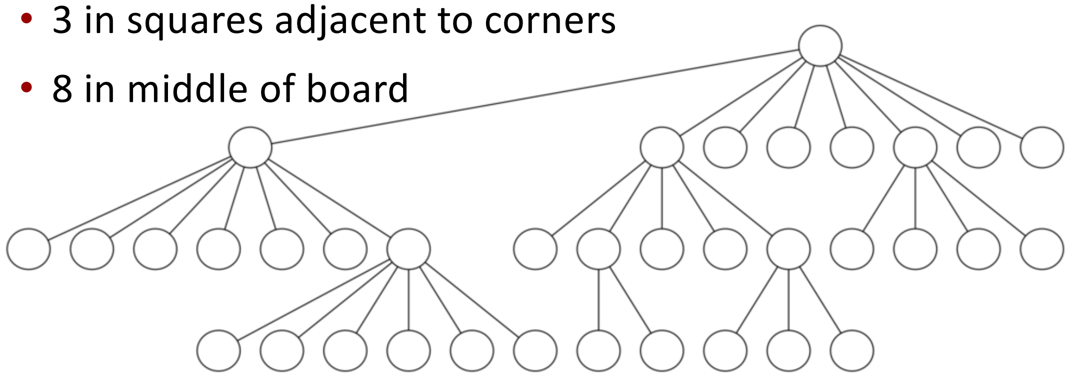
Knight's Tour - Analysis

- Root is starting point of search tree
- Then checks each move knight can make
  - 2 legal moves in corner
  - 3 in squares adjacent to corners
  - 8 in middle of board



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Knight's Tour - Analysis

- Figure shows number of possible moves on board
- Next level of tree has again 2 – 8 next possible moves
- Number of possible positions to examine corresponds to number of nodes in search tree

2	3	4	4	4	4	3	2
3	4	6	6	6	6	4	3
4	6	8	8	8	8	6	4
4	6	8	8	8	8	6	4
4	6	8	8	8	8	6	4
4	6	8	8	8	8	6	4
3	4	6	6	6	6	4	3
2	3	4	4	4	4	3	2

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Knight's Tour - Analysis

- Number of nodes in binary tree is $2^{N+1}-1$
- Number much larger for tree with up to 8 nodes
- Use average branch factor to estimate number of child nodes: $k^{N+1}-1$, *k* is average branching factor
- Example:
  - 5 x 5 board, tree is 25 levels deep => *N=24*
  - *k=3.8* => $3.8^{25}-1 = 3.12 * 10^{14}$

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Knight's Tour - Analysis

- Way to speed up 8 x 8 case => runs in less than 1 second
- `orderbyAvail` will be called used instead of `u.getConnections` (shown in previous code)
- Line 10 is critical one, it ensures to select vertex that has *fewest* available moves
- But why not select node that has *most* available moves?

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Knight's Tour - Analysis

```
def orderByAvail(n):
    resList = []
    for v in n.getConnections():
        if v.getColor() == 'white':
            c = 0
            for w in v.getConnections():
                if w.getColor() == 'white':
                    c = c + 1
            resList.append((c,v))
    resList.sort(key=lambda x: x[0])
    return [y[1] for y in resList]
```

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Knight’s Tour - Analysis

- Problem with using vertex with most available moves => tends to have knight visit middles squares early on
  - Easy for night to get stranded on one side of board and cannot reach other side.
- Visiting squares with fewest available moves first pushes knight to visit squares around edges
- Using intuition is called *heuristic!*

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General Depth First Search

- Implementation extends graph class by adding:
  - Time instance variable and methods `dfs` and `dfsvisit`
  - `dfs` method iterates over all vertices in graph calling `dfsvisit` on white nodes
  - This ensures all nodes in graph are considered and no vertices are left out of depth first forest

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General Depth First Search

```
from Graph import Graph, Vertex
class DFSGraph(Graph):
    def __init__(self):
       super().__init__()
       self.time = 0

    def dfs(self):
        for aVertex in self:
            aVertex.setColor('white')
          aVertex.setPred(-1)
        for aVertex in self:
          if aVertex.getColor() == 'white':
             self.dfsvisit(aVertex)

    def dfsvisit(self,startVertex):
        startVertex.setColor('gray')
       self.time += 1
        startVertex.setDiscovery(self.time)
        for nextVertex in startVertex.getConnections():
            if nextVertex.getColor() == 'white':
                nextVertex.setPred(startVertex)
                self.dfsvisit(nextVertex)
        startVertex.setColor('black')
       self.time += 1
        startVertex.setFinish(self.time)
```


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General Depth First Search

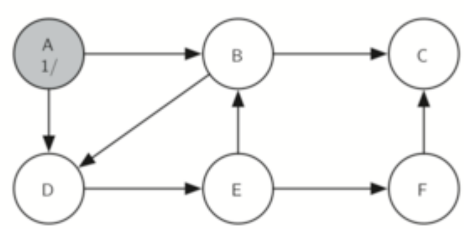
- DFS method starts with single vertex `startVertex` and explores all neighboring white vertices as deeply as possible
- `dfsvisit` is almost identical to `bfsexcept`
- `dfsvisit` uses a stack where `bfsexcept` uses queue
  - Not visible in code but implicit of `dfsvisit`

General Depth First Search

- Following sequence of figures illustrates DFS in action
- Dotted lines indicate checked edges but node on other end of edge has already been added to DFS tree
- In the code this is realized by checking that color of the other node is non-white

General Depth First Search

- Search begins at vertex A
- Since all vertices are white algorithm visits vertex A
  1. Set color of vertex A gray => vertex is being explored
  2. Discovery time is set to 1
  3. Neighbors B and D need to be visited as well
  4. Arbitrary decision to visit adjacent nodes in alphabetic order



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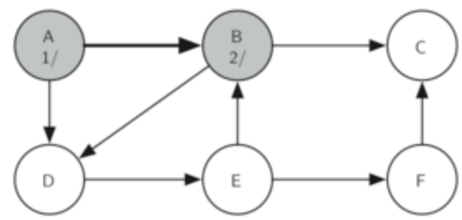
General Depth First Search

- Vertex B is visited next
  1. Its color is set to gray
  2. Discovery time is set to 2
  3. B is adjacent to C and D
  4. Visit vertex C next



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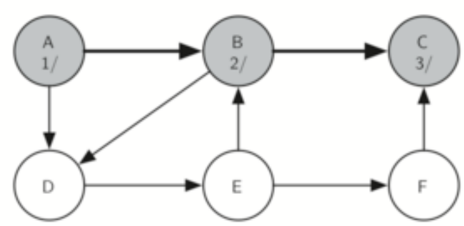
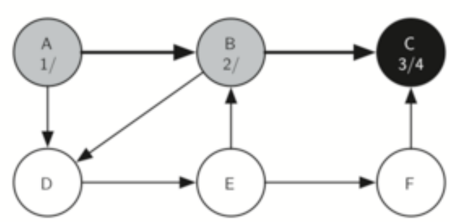
General Depth First Search

- Visiting C brings alg. to end of branch of tree
  1. Color node gray and set discovery time to 3
  2. No adjacent vertices to C
  3. Color vertex black, set finish time to 4





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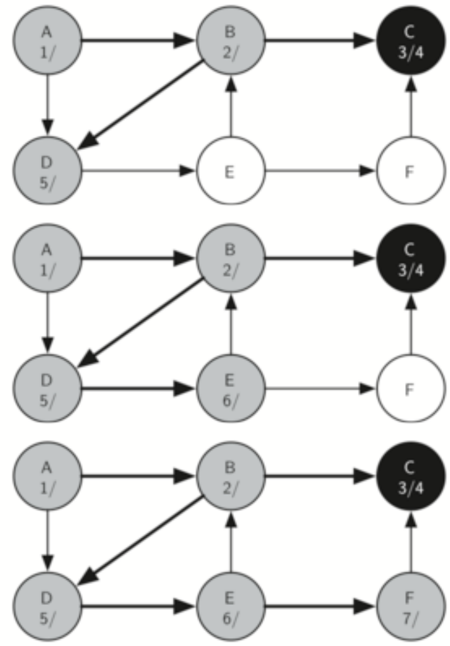
General Depth First Search

- Now return to B and explore nodes adjacent to it
- Only addition vertex is D
  - Visit D and continue search
  - Results in exploring E, which has adjacent vertices B and F
  - B is already colored, thus explore F



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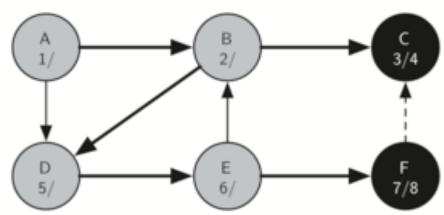
General Depth First Search

- F has only adjacent vertex C
  - C already colored black
  - Nothing else to explore
  - Reached end of branch
- Algorithm works its way back to first node
  - setting finish times and
  - coloring vertices black



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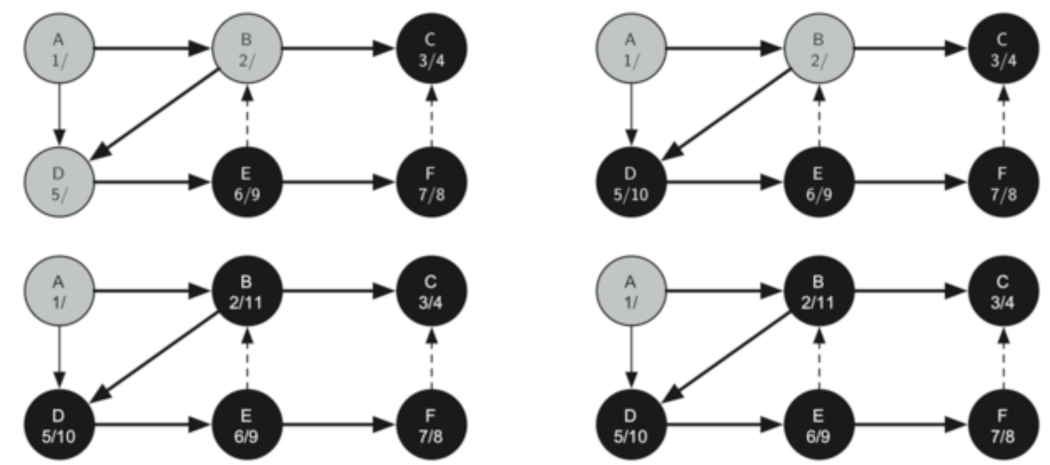
General Depth First Search



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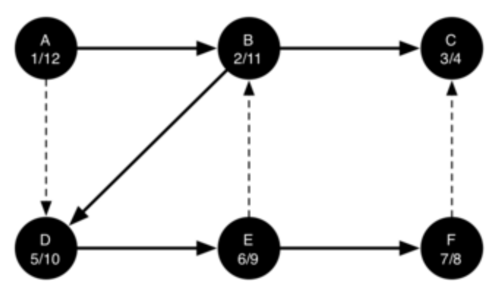
General Depth First Search

- Start and finishing times are called *parentheses property*
- All children of particular node in DFS
  - Have later discovery time than parent
  - Have earlier finish time than parent
- Figure shows final tree constructed by DFS algorithm



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General Depth First Search

- General running time:
  - Loops in dfs run in *O(V)*, since executed once for each vertex in graph
  - Since dfsvisit only called recursively if vertex is white, loop will execute max. once for every edge in graph => *O(E)*
- Total time for DFS is *O(V+E)*

Topological Sorting

- Demonstrate that almost anything can be turned into a graph problem
- Consider problem of stirring up batch of pancakes
- Recipe: 1egg, 1 cup of pancake mix, 1 tablespoon oil and ¾ cup of milk
- Heat griddle, mix all ingredients together, and spoon mix onto hot griddle
- When pancakes start bubbling, turn them over
- Heat up syrup

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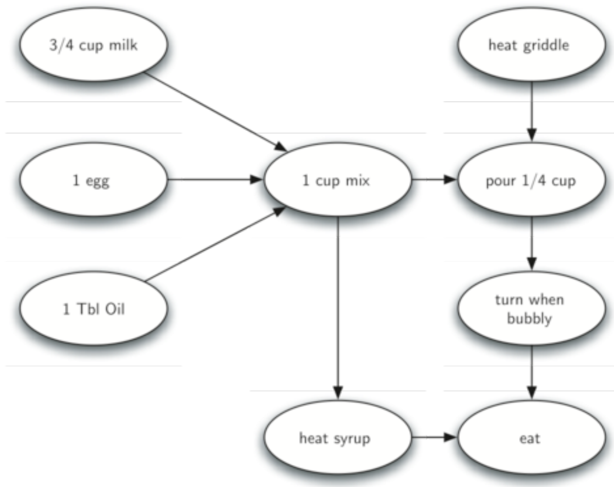
Topological Sorting

- Here the process is illustrated as a graph



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Topological Sorting

- Problem: Know what to do first
- Start by heating griddle or adding any of ingredients to pancake mix
- To make that decision we turn to algorithm called *topological sort*

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Topological Sorting

- Topological sort takes DAG and produces linear ordering of all vertices such that
  - If graph contains edge *(v,w)* then vertex *v* comes before vertex *w.*
- Other examples besides pancakes:
  - project schedules
  - Multiplying matrices

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Topological Sorting

- Algorithm for Topological Sort (adaptation of DFS):
  1. Call `dfs(g)` for some graph *g.* Main reason, call finish times for each vertex
  2. Store vertices in a list in decreasing order of finish time
  3. Return the ordered list as the result of the topological sort

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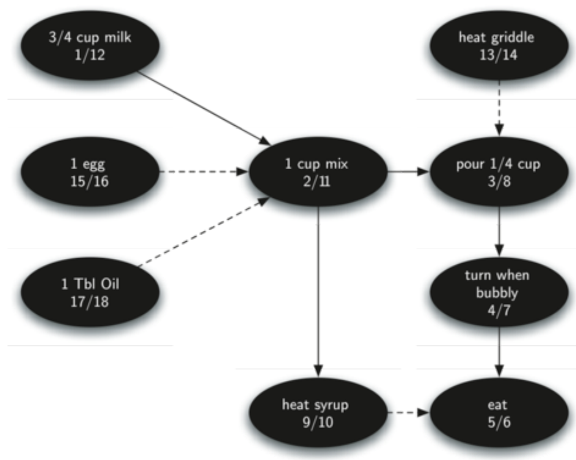
Topological Sorting

- Tree constructed by DFS



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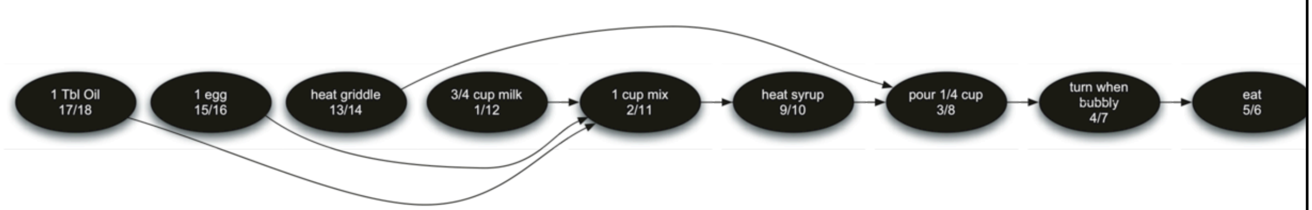
Topological Sorting

- Result of applying topological sorting to graph
- Now we know exactly order in which to make pancakes



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Next Steps

- Next lecture on Tuesday: State Machines

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